Electronic Supplement to

Framework for regional seismic risk assessments of groups of tall buildings

by James Bantis, Pablo Heresi, Alan Poulos, and Eduardo Miranda

1 Ground motion modeling

Response spectra at all sites are sampled from a multivariate normal distribution. Let s be the number of sites and p the number of periods of the response spectra, then a vector of spectral accelerations at all sites and periods, $\boldsymbol{Sa} \in \mathbb{R}^{sp}$, can be sampled as:

$$\ln \boldsymbol{Sa} = \boldsymbol{\mu}_{\ln Sa} + \boldsymbol{\varepsilon}$$

where $\boldsymbol{\mu}_{\ln Sa} \in \mathbb{R}^{sp}$ are the mean logarithms obtained from a Ground Motion Model (GMM) and $\boldsymbol{\varepsilon} \in \mathbb{R}^{sp}$ is a sample from a zero-mean multivariate normal distribution:

$$\boldsymbol{\varepsilon} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}) \tag{1}$$

The required covariance matrix, $\boldsymbol{\Sigma} \in \mathbb{R}^{sp \times sp}$, can be constructed as a block matrix:

$$\boldsymbol{\Sigma} = \begin{bmatrix} \boldsymbol{\Sigma}_{11} & \boldsymbol{\Sigma}_{12} & \cdots & \boldsymbol{\Sigma}_{1p} \\ \boldsymbol{\Sigma}_{21} & \boldsymbol{\Sigma}_{22} & \cdots & \boldsymbol{\Sigma}_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{\Sigma}_{p1} & \boldsymbol{\Sigma}_{p2} & \cdots & \boldsymbol{\Sigma}_{pp} \end{bmatrix} \tag{2}$$

Each block, $\boldsymbol{\Sigma}_{ij} \in \mathbb{R}^{s \times s}$, of the covariance matrix has the following form:

$$\boldsymbol{\Sigma}_{ij} = \rho_0(T_i, T_j) \left[\boldsymbol{\tau}(T_i)\boldsymbol{\tau}(T_j)^T + \boldsymbol{R}_{ij} \odot \boldsymbol{\sigma}(T_i)\boldsymbol{\sigma}(T_j)^T \right], \quad \forall\, i, j = 1, \ldots, p \tag{3}$$

where $\boldsymbol{\tau}(T_i)$ and $\boldsymbol{\sigma}(T_i) \in \mathbb{R}^s$ is a vector of between-event and within-event standard deviation terms, respectively, for spectral accelerations at all sites and period T_i; $\odot$ denotes an element-wise product; $\rho_0(T_i, T_j)$ is the spectral correlation between periods T_i and T_j, which in this study was computed using the model proposed by Baker and Jayaram (2008); and $\boldsymbol{R}_{ij} \in \mathbb{R}^{s \times s}$ is a matrix of within-event spatial correlations between periods T_i and T_j, with each of its entries computed as:

$$(\boldsymbol{R}_{ij})_{k,l} = \rho_W(\Delta_{kl}, \max\{T_i, T_j\}), \quad \forall\, k, l = 1, \ldots, s \tag{4}$$

where Δ_{kl} is the distance between sites k and l; and $\rho_W(\Delta, T)$ is the within-event spatial correlation of spectral accelerations at period T of sites separated by a distance Δ, which for this study was obtained using the model developed by Heresi and Miranda (2019).

Note that the proposed sampling scheme returns a vector $\boldsymbol{Sa}$ where the first s entries correspond to spectral accelerations for the first period at all sites, the second s entries correspond to the second period, and so on. A similar scheme can be defined where each block of the covariance matrix would correspond to spectral accelerations for all periods at a pair of sites.

2 Site response analysis

2.1 Inverse random vibration theory

The Inverse Random Vibration Theory (IRVT) portion of the site response analysis procedure is used to transform a rock outcrop response spectrum into a Fourier amplitude spectrum (FAS). This transformation is conducted using the procedure described by Rathje et al. (2005):

$$FAS(f_n) = \sqrt{\left(\frac{1}{f_n(\frac{\pi}{4\zeta} - 1)}\right)\left(\frac{T_d\,[Sa(f_n)]^2}{2\,[p(f_n)]^2} - \int_0^{f_n} |FAS(f)|^2 df\right)} \tag{5}$$

where f_n is the n-th frequency; $FAS(f_n)$ is the FAS at f_n; ζ is the damping ratio of the inputted response spectrum; T_d is the ground motion duration, taken here as the D_{5-95} significant ground motion duration; $Sa(f_n)$ is the rock outcrop response spectrum at f_n; and $p(f_n)$ is the peak factor at f_n, which is computed with the method proposed by Davenport (1964). Because the FAS also appears on the right side of Equation (5), this equation is first used at the lowest frequency, where the integral can be assumed to be zero, and is then used at successively higher frequencies. Moreover, the peak factors also depend on the FAS. Thus, Equation (5) is applied in two steps, first assuming that the peak factor is 2.5 at all frequencies, then using the peak factors that result from the FAS of the first step. For the San Francisco case study, the D_{5-95} significant ground motion duration is obtained from the Afshari and Stewart (2016) ground motion duration model, which depends on the moment magnitude, Joyner-Boore distance, and V_{s30} of the rock site.

2.2 Sampling the soil column dynamic parameters

The four parameters needed to obtain the transfer function representing the ratio of the FAS at the ground surface to the FAS at bedrock are δ of the continuous shear beam modeling the soil column, modal damping ratios of the soil column, the predominant period of the soil deposit, and the depth to bedrock which is assumed to be known. The predominant period of the soil can be computed using a simplified equation from Kramer (1996), who showed that, for the case of uniform damped soil on rigid rock, a linear elastic approach leads to the predominant period of the soil deposit, T_g, being:

$$T_g = \frac{4D_b}{V_s^\star} \tag{6}$$

where D_b is the depth to bedrock at the site of interest and $V_s^\star$ is the equivalent homogeneous shear wave velocity of the uniform soil layer (Garcia-Suarez & Asimaki, 2020). Equation (6) is simplified because in reality there are many soil layers underlain each site with changing shear wave velocities. However, the simplified equation can be fairly accurate if $V_s^\star$ of the one-layer soil profile is the average shear wave velocity of the entire true multi-layered soil profile. Because the soil profiles at each site in a regional analysis will not be known, the value of $V_s^\star$ at each site cannot be calculated. Therefore, a single value of $V_s^\star$ for all sites in a region is used, which is obtained from all the records of previous earthquakes in the region of interest. At each site with available records from previous earthquakes, the value of T_g is estimated using one or multiple methods suggested in Bantis and Miranda (2024). From there, $V_s^\star$ at each site can be back-calculated from the known depth to bedrock and the estimated T_g. Then the average of all the computed $V_s^\star$ values from the individual sites is used as the single $V_s^\star$ value for the region of interest. Once this value is computed, the predominant period of the soil deposit is sampled as:

$$T_g = \frac{4D_b}{V_s^\star} \exp(\varepsilon_{Tg}) \tag{7}$$

where ε_{Tg} is a sample from a zero-mean Gaussian distribution with a standard deviation of 0.1. For the San Francisco case study, $V_s^\star$ was calculated to be 338 m/s based on the 10 sites listed in Table 1. Table 1 additionally shows the depth to bedrock, identified T_g, and back-calculated $V_s^\star$ for each site based on system identification from the available records at each site.

Table 1: Calculated $V_s^\star$ values for the San Francisco case study.

Site	D_b (m)	T_g (s)	$V_s^\star$ (m/s)
1	78.3	0.97	323.0
2	61.3	0.75	326.8
3	41.8	0.47	355.4
4	23.5	0.26	361.1
5	27.1	0.34	319.1
6	75.9	0.87	348.9
7	75.6	0.86	351.6
8	82.3	0.99	332.5
9	67.0	0.84	319.1
10	28.0	0.33	339.4

To obtain δ and the modal damping ratios of the soil column, earthquake records from downhole array sites in the region of interest are used. At each site with recordings, system identification with the shear beam model and linear elastic response history analysis is used to find the values of δ, T_g, and modal damping ratios that best fit the recorded ground motions throughout the depth of the soil profile. The δ value for all sites in the regional analysis is then taken as the geometric mean of the δ values from sites used in the system identification. Modal damping ratios for the soil column are sampled from a lognormal probability distribution, where the median and dispersion (logarithmic standard deviation) for each mode are the median and dispersion of that mode's damping ratios over all the sites considered in the system identification. For the San Francisco case study, the following five sites with downhole arrays in the San Francisco Bay Area were used in the system identification: Treasure Island, Embarcadero Plaza, Bessie Carmichael School, Levi Plaza, and Foster City – Redwood Shores. Table 2 gives the resulting parameters for the case study, considering six modes for the soil model transfer function. In this case, the damping ratio parameters for all higher modes are the same.

2.3 Sampling the rock column dynamic parameters

The parameters of the rock column are obtained similarly to those of the soil column but with two differences. The first difference is that the bedrock records from previous earthquakes are paired with records at rock outcrop sites from the same earthquake. For the case study, the following

Table 2: Selected δ and ζ parameters of the soil model for the San Francisco case study.

Parameter	Value
δ	0.01
ζ_1 median	0.08
ζ_1 dispersion	0.4
$\zeta_{2\to 6}$ median	0.06
$\zeta_{2\to 6}$ dispersion	0.4

rock outcrop sites were used: Russian Hill, Fort Mason Hill, Yerba Buena Island, Rincon Hill, and Potrero Hill. The second difference is that the predominant period of the rock column is sampled using a power law equation:

$$T_g = mD_b^n \exp(\varepsilon_{Tg}) \tag{8}$$

where m and n are parameters fitted with the estimated period of the rock column and the depth to bedrock of sites with earthquake records, and ε_{Tg} is a sample from a zero-mean truncated Gaussian probability distribution with a range from -2 to 2 and a standard deviation calculated from the empirical data. Table 3 shows the calculated values of δ, m, n, the standard deviation of T_g (σ_{Tg}), and the median and dispersion of both the first and higher modal damping ratios used in the case study.

Table 3: Selected parameters of the rock model for the San Francisco case study.

Parameter	Value
δ	0.33
ζ_1 median	0.08
ζ_1 dispersion	0.5
$\zeta_{2\to 6}$ median	0.04
$\zeta_{2\to 6}$ dispersion	0.5
m	0.084
n	0.283
σ_{Tg}	0.39

2.4 Random vibration theory

To conduct the Random Vibration Theory (RVT) portion of the site response analysis procedure, the Davenport (1964) peak factor is used in conjunction with the D_{5-95} significant ground motion duration sampled from a ground motion duration model. For the San Francisco case study, the Afshari and Stewart (2016) ground motion duration model for shallow crustal earthquakes in active tectonic regions is used. The moment magnitude, Joyner-Boore distance, V_{s30} of the rock site, and strike-slip fault type are used as inputs to sample a D_{5-95} significant ground motion duration for each site. For more information on how to conduct the RVT, the reader is referred to Bantis and Miranda (2023).

3 Characterization of dynamic properties of buildings

The dynamic characteristics of each building are fully defined by four parameters: the fundamental period (T_1), the first modal damping ratio (ζ_1), the lateral stiffness ratio (α), and parameter δ. This section describes how these parameters are obtained for three cases: (1) instrumented buildings with records from previous earthquakes, (2) instrumented buildings with ambient vibrations, and (3) buildings where only the structural system and height are known.

3.1 Buildings with earthquake records

For buildings with recorded responses throughout their heights from previous earthquakes, system identification using the continuous coupled shear and flexural beam model is used. Displacement and acceleration response histories, as well as response spectra at different heights of the building, are estimated by a linear elastic modal response history analysis of the continuous coupled model with the ground motion at the base of the building as input. The parameters T_1, ζ_1, δ, and α are obtained for each building and each principal orientation as those that minimize the difference between estimated and recorded responses. Additionally, the modal damping ratios for higher modes can be obtained directly from system identification rather than the Cruz and Miranda (2017) mentioned in the main manuscript.

For the San Francisco case study, eleven buildings had recorded responses throughout their heights from previous earthquakes. In the interest of space, Table 4 gives the values of the parameters determined using system identification for the four buildings that recorded the 1989 M_W 6.9 Loma Prieta earthquake. These buildings are listed as buildings A, B, C, and D in Table 4. Six modes of vibration were considered.

Table 4: Dynamic properties of buildings with Loma Prieta earthquake records

Parameters	Bldg. A		Bldg. B		Bldg. C		Bldg. D	
	NS	EW	NS	EW	NS	EW	NS	EW
T_1 (s)	2.28	3.20	5.27	6.23	6.09	4.96	3.56	3.67
α	30	30	15	30	9	18	5	21
δ	1	1	0.55	0.45	1	0.55	0.75	0.15
ζ_1	0.04	0.025	0.01	0.015	0.01	0.015	0.011	0.01
ζ_2	0.08	0.03	0.015	0.012	0.014	0.047	0.045	0.06
ζ_3	0.06	0.07	0.01	0.023	0.04	0.077	0.09	0.022
ζ_4	0.11	0.02	0.01	0.009	0.05	0.082	0.05	0.08
ζ_5	0.09	0.015	0.01	0.01	0.03	0.05	0.03	0.02
ζ_6	0.07	0.05	0.01	0.011	0.07	0.05	0.01	0.02

3.2 Buildings with ambient vibrations

For buildings that recorded ambient vibrations, this information can be used to reduce the uncertainty when estimating the fundamental period of the building. Once the fundamental period from ambient vibrations (T_{Ambient}) is estimated, the fundamental period of the building is sampled using the following equation:

$$T_1 = \psi T_{\text{Ambient}} \tag{9}$$

where ψ is the ratio of the fundamental period of the building during an earthquake to the fundamental period of the building from ambient vibrations, and is sampled using the following equation:

$$\psi = \exp(\mu_{\ln\psi} + \varepsilon_{\ln\psi}\sigma_{\ln\psi}) \tag{10}$$

where $\varepsilon_{\ln\psi}$ is sampled from a standard normal distribution truncated between -3 and 3, and $\mu_{\ln\psi}$ and $\sigma_{\ln\psi}$ depend on the type of structural system and were fitted with buildings where the fundamental period was estimated using system identification during an earthquake and also with ambient vibrations. Table 5 presents these parameters based on the structural type of the building.

Table 5: $\mu_{\ln\psi}$ and $\sigma_{\ln\psi}$ parameters.

Structural System	$\mu_{\ln\psi}$	$\sigma_{\ln\psi}$
Reinforced Concrete Shear Wall	0.2442	0.1167
Steel Braced Frame	0.2136	0.0929
Steel Moment Frame	0.2136	0.0929
Mixed	0.2442	0.1167
Unknown	0.2442	0.1167

The three other parameters are estimated using only the height of the building and its structural system. Because the α parameter controls the degree of participation of overall flexural and shear deformations, it is related to the type of structural system of the building. Thus, α is sampled using a uniform probability distribution where the upper and lower limits depend on the type of structural system. System identification using the continuous coupled model and modal response history analysis was performed on many buildings in order to gauge the range of α for each structural type, resulting in the upper and lower limits of α shown in Table 6.

Table 6: Upper and lower limits of the uniform probability distribution used to sample α.

Structural System	α_{lower}	α_{upper}
Reinforced Concrete Shear Wall	0	5
Steel Braced Frame	0	5
Steel Moment Frame	15	40
Mixed	5	20
Unknown	0	40

Parameter δ is taken as deterministic and as a function of building height based on work from Miranda and Taghavi (2005):

$$\delta = \max\{0.01, 0.35 - 0.001H\} \tag{11}$$

where H is the building height in meters. The δ parameter does not need to be sampled because, for realistic δ values of buildings, the period ratios and effective mode shapes do not change significantly.

Finally, the first modal damping ratio at each of the two perpendicular principal directions of the building is sampled using the following equation:

$$\zeta_1 = \exp(a_\zeta + b_\zeta \ln(H) + \varepsilon_\zeta \sigma_\zeta) \tag{12}$$

where a_ζ, b_ζ, and σ_ζ are parameters shown in Table 7 that depend on the building structural system and are based on the work of Cruz (2017); and ε_ζ is one of the components of a sample from a bivariate normal probability distribution with zero mean, unit standard deviation, and a covariance matrix given by a correlation matrix between the first modal damping ratio in the transverse direction of the building and the first modal damping ratio in the longitudinal direction of the building. These correlation matrices correspond to the bottom right 2x2 blocks of the matrices shown in Table 8, which depend on the building structural system and are based on the work of Cruz (2017). The full 4x4 correlation matrices shown in Table 8 correspond to all correlations between fundamental periods in the transverse and longitudinal directions and the first modal damping ratios in the transverse and longitudinal directions.

Table 7: Parameters used to sample ζ_1.

Structural System	a_ζ	b_ζ	σ_ζ
Reinforced Concrete Shear Wall	-1.9880	-0.3315	0.4354
Steel Braced Frame	-1.8606	-0.4393	0.2855
Steel Moment Frame	-1.2828	-0.5215	0.3173
Mixed	-0.7604	-0.6765	0.4689
Unknown	-1.5161	-0.4419	0.4757

For the San Francisco case study, ten buildings studied by Byerly et al. (1931) fell into this ambient vibration category. The periods estimated by ambient vibrations for these buildings are shown in Table 9.

3.3 Buildings with limited information

The final way of sampling the four major parameters is for buildings with limited information where only the structural system and height are known. For these buildings, α and δ are determined in the same way as for the buildings with ambient vibrations. Unlike the buildings with ambient vibrations where the fundamental period of the building and first modal damping ratio could be determined separately, for buildings with limited information, T_1 and ζ_1 are determined simultaneously. The equations to simulate these parameters, analogous to Equation (12), are as follows:

$$\zeta_1 = \exp(a_\zeta + b_\zeta \ln(H) + \varepsilon_{T\zeta} \sigma_\zeta) \tag{13}$$

$$T_1 = \exp(a_T + b_T \ln(H) + \varepsilon_{T\zeta} \sigma_T) \tag{14}$$

where a_ζ, b_ζ, σ_ζ, a_T, b_T, and σ_T are parameters that depend on the structural system of the building and are based on the work of Cruz (2017). Table 7 and Table 10 show the values of these parameters

Table 8: Correlation between T_1 and ζ_1 in the transverse and longitudinal directions of the building.

Structural System	T_1		ζ_1	
	Transverse	Longitudinal	Transverse	Longitudinal
Reinforced Concrete Shear Wall	1.0000	0.3727	0.0245	0.0970
Reinforced Concrete Shear Wall	0.3727	1.0000	0.0341	-0.1421
Reinforced Concrete Shear Wall	0.0245	0.0341	1.0000	0.4843
Reinforced Concrete Shear Wall	0.0970	-0.1421	0.4843	1.0000
Steel Braced Frame	1.0000	0.8926	-0.2524	-0.1159
Steel Braced Frame	0.8926	1.0000	-0.1981	-0.0422
Steel Braced Frame	-0.2524	-0.1981	1.0000	0.4058
Steel Braced Frame	-0.1159	-0.0422	0.4058	1.0000
Steel Moment Frame	1.0000	0.9678	0.2911	0.3238
Steel Moment Frame	0.9678	1.0000	0.3573	0.2866
Steel Moment Frame	0.2911	0.3573	1.0000	0.1440
Steel Moment Frame	0.3238	0.2866	0.1440	1.0000
Mixed	1.0000	0.7860	-0.2812	-0.3119
Mixed	0.7860	1.0000	0.0620	-0.0524
Mixed	-0.2812	0.0620	1.0000	0.5639
Mixed	-0.3119	-0.0524	0.5639	1.0000
Unknown	1.0000	0.7773	0.1846	0.4138
Unknown	0.7773	1.0000	0.2965	0.5635
Unknown	0.1846	0.2965	1.0000	0.4853
Unknown	0.4138	0.5635	0.4853	1.0000

based on the structural system. $\varepsilon_{T\zeta}$ is the corresponding component of a sample from a multivariate normal probability distribution with zero mean, unit standard deviation, and a covariance matrix given by the 4x4 correlation matrix given in Table 8. Note that, due to the order in the correlation matrix, the first two components of the sampled vector are used as $\varepsilon_{T\zeta}$ for the fundamental periods, and the last two components are used for the first modal damping ratios. All components of the sample are truncated between -2 and 2. For the San Francisco case study, 159 buildings fell into this sampling category.

4 Structural response

Variable ρ_{ij} is the correlation between total accelerations of two modes of vibration i and j. Variable ρ_{jg} is the correlation between the ground acceleration and total modal acceleration of the jth mode. Based on empirical results from response history analyses, Taghavi and Miranda (2006) fitted correlation equations for both ρ_{ij} and ρ_{jg}. These equations are updated here and are as follows:

$$\rho_{ij} = 1 - 1.1\exp(-\omega_{min}^{1.23}(0.0806\zeta_{mean} + 0.0036)) \tag{15}$$

Table 9: Periods of vibration of ten buildings used in the San Francisco case study estimated using ambient vibrations.

Building	T_{Ambient}	
	Transverse	Longitudinal
1	1.20	1.40
2	1.80	1.85
3	1.71	1.89
4	1.33	1.48
5	0.95	1.27
6	1.32	1.34
7	1.49	1.82
8	1.41	1.64
9	1.50	1.28
10	0.65	0.65

Table 10: Parameters used to sample T_1.

Structural System	a_T	b_T	σ_T
Reinforced Concrete Shear Wall	-3.8570	0.9684	0.3728
Steel Braced Frame	-3.4514	0.9290	0.2865
Steel Moment Frame	-3.1366	0.9297	0.3081
Mixed	-5.5649	1.4528	0.4952
Unknown	-3.8626	1.1143	0.3902

$$\rho_{jg} = 1 - 1.2\exp(-\omega_j(0.2371\zeta_j + 0.0124)) \tag{16}$$

where ω_j is the circular frequency of the j-th mode, ζ_j is the damping ratio of the j-th mode, ω_{min} is the minimum circular frequency of modes i and j, and ζ_{mean} is the mean damping ratio of modes i and j.

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